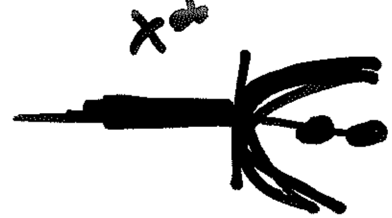
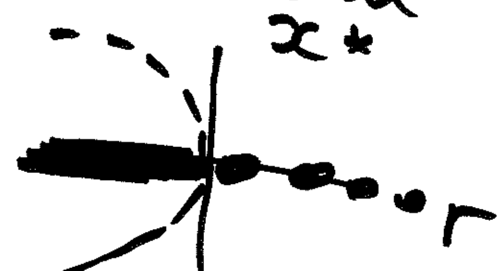
$$\frac{d}{dt}x(t) = rx - x^3$$



- supercritical

$$\frac{d}{dt}x(t) = rx + x^3$$

- subcritical



$$x(t) = \pm\sqrt{r}\cdot\frac{1}{\sqrt{(1+r/x_0)e^{-rt}-1}}$$

$$\frac{d}{dt}x(t) = rx + x^3 - x^5$$

"safe guard"

$a\dot{x} = bx + cx^3 - dx^5;\quad a, b, c, d > 0$ 2

$x = \alpha y;$

$t = \beta \cdot T$

$[x] = [\alpha]$; dimensions are equal

$[y] = 1$ - dimensionless

$[t] = [\beta]$

$[T] = 1$

$$\frac{d}{dt} = \frac{dT}{dt}\frac{d}{dT} = \frac{1}{\beta}\frac{d}{dT}$$

$$\frac{d}{dT}y = Ry + y^3 - y^5$$

$$\left(\alpha \frac{a}{\beta} \cdot \frac{d}{dT} y = b\alpha y + c\alpha^3 y^3 - d\alpha^5 y^5\right)\frac{\beta}{\alpha a}$$

$$y \equiv y(T)$$

$$\frac{d}{dT} y(T) = \overbrace{\frac{\beta b}{a}}^{R} y + \overbrace{\frac{c\alpha^3\beta}{\alpha a}}^{1} y^3 - \overbrace{\frac{d\alpha^5\beta}{\alpha a}}^{1} y^5$$

$$c\alpha^2\beta = a$$

$$d\alpha^4\beta = a$$

$$\frac{d\alpha^2}{c} = 1 \; ; \quad \alpha = \sqrt{\frac{c}{d}}$$

$$\boxed{\frac{\beta b}{a} = \frac{bd}{c^2} = R}$$

$$\beta = \frac{a}{c\alpha^2} = \frac{a}{c}\,\frac{d}{c} = \frac{ad}{c^2}$$

$$\dot{x} = rx + x^3 - x^5$$

$$x^* = \emptyset; \quad x^* = \pm\sqrt{\frac{1 \pm \sqrt{1+4r}}{2}}$$

$1 + 4r > \emptyset, \quad r > -1/4$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(rx + x^3 - x^5\right) = r + 3x^2 - 5x^4$$

$$x_{1,2} = \pm\sqrt{\frac{1 - \sqrt{1+4r}}{2}} \qquad \emptyset > r > -1/4$$

$$x_{3,4} = \pm\frac{1}{\sqrt{2}}\sqrt{1 + \sqrt{1+4r}}$$

$$x_{1,2}^2 = \frac{1-\sqrt{\ }}{2}, \quad \sqrt{\ } \equiv \sqrt{1+4r}$$

$$\left.\frac{\partial}{\partial x} f\right|_{x=x_{1,2}} = r + \frac{3}{2}\left(1-\sqrt{\ }\right) - \frac{5}{4}\left(1+1+4r-2\sqrt{\ }\right)$$

$$= r + \frac{3}{2} - \frac{3}{2}\sqrt{\ } - \frac{5}{2} - 5r + \frac{5}{2}\sqrt{\ } =$$

$$= -4r - 1 + \sqrt{\ }$$

$$= -(1+4r) + \sqrt{1+4r} = \boxed{\left.\frac{\partial f}{\partial x}\right|_{x=x_{1,2}} > 0, \quad x_{1,2}}$$

$$-\frac{1}{4} < r < 0$$

unstable

$$x_{1,2} = \pm \sqrt{\frac{1-\sqrt{1+4r}}{2}} \quad Re$$

$$\boxed{-\tfrac{1}{4} < r < \emptyset}$$

$r = \emptyset, \; x_1 = x_2 = \emptyset$

$r = -\tfrac{1}{4}; \; x_{1,2} = \pm \sqrt{\tfrac{1}{2}}$

$$\frac{dx_{1,2}}{dr} = -\frac{1}{\sqrt{2}\sqrt{1+4r}\sqrt{1-\sqrt{1+4r}}}$$

$r = \emptyset, \; \frac{dx}{dr} = \infty$

$r = -\tfrac{1}{4}; \; \frac{d\,i}{dx}$

$\frac{dx_{12}}{dr} = \infty, \; r = \emptyset, -\tfrac{1}{4}$

$$x_{3,4}^2 = \frac{1+\sqrt{1+4r}}{2};$$

$$\frac{\partial f}{\partial x} = r + 3x^2 - 5x^4$$

$$= r + \frac{3}{2}(1+\sqrt{\ }) - \frac{5}{4}(1+1+4r+2\sqrt{\ })$$

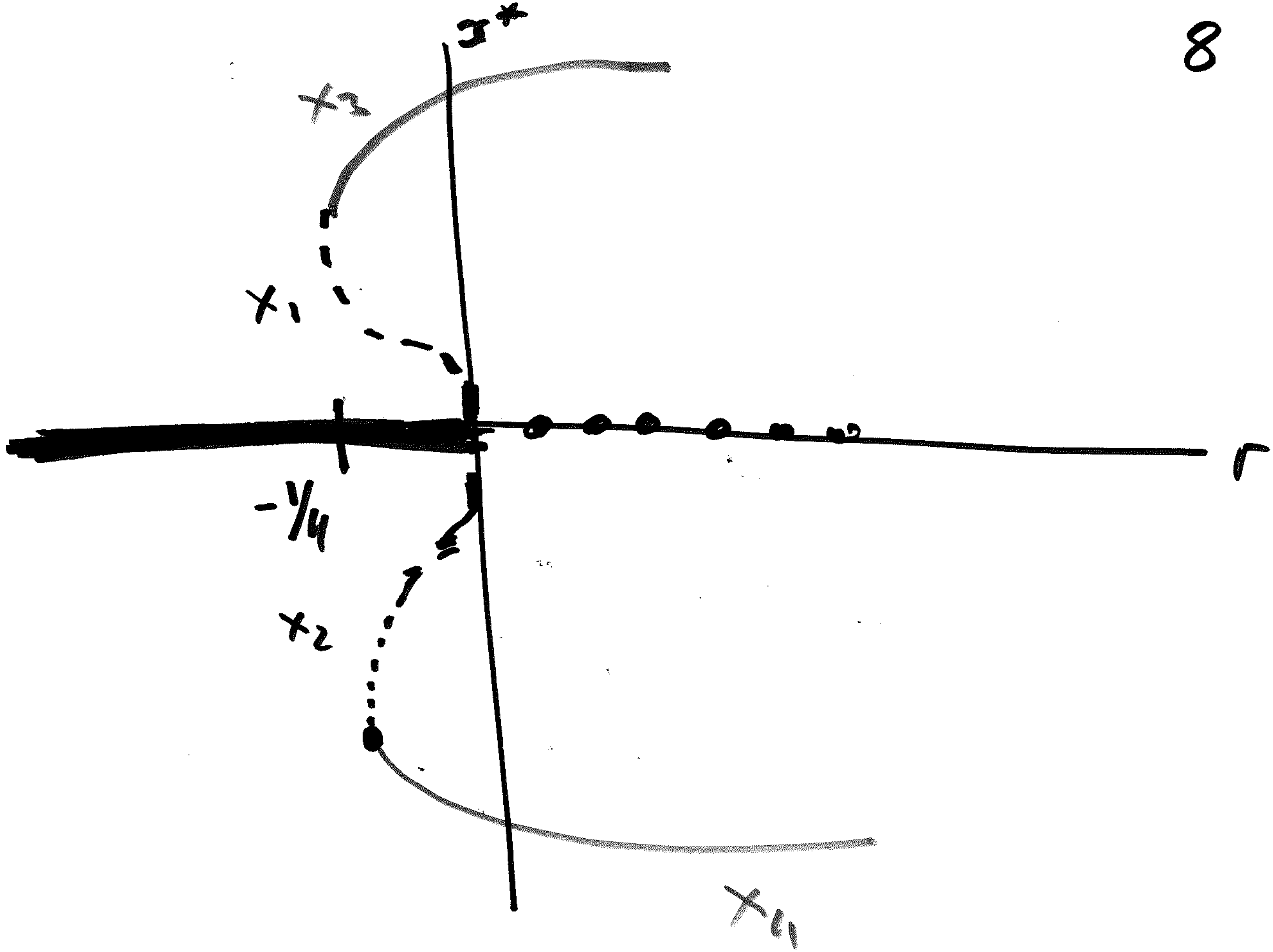
$$= r + \frac{3}{2} + \frac{3}{2}\sqrt{\ } - \frac{5}{2} - 5r - \frac{5}{2}\sqrt{\ }$$

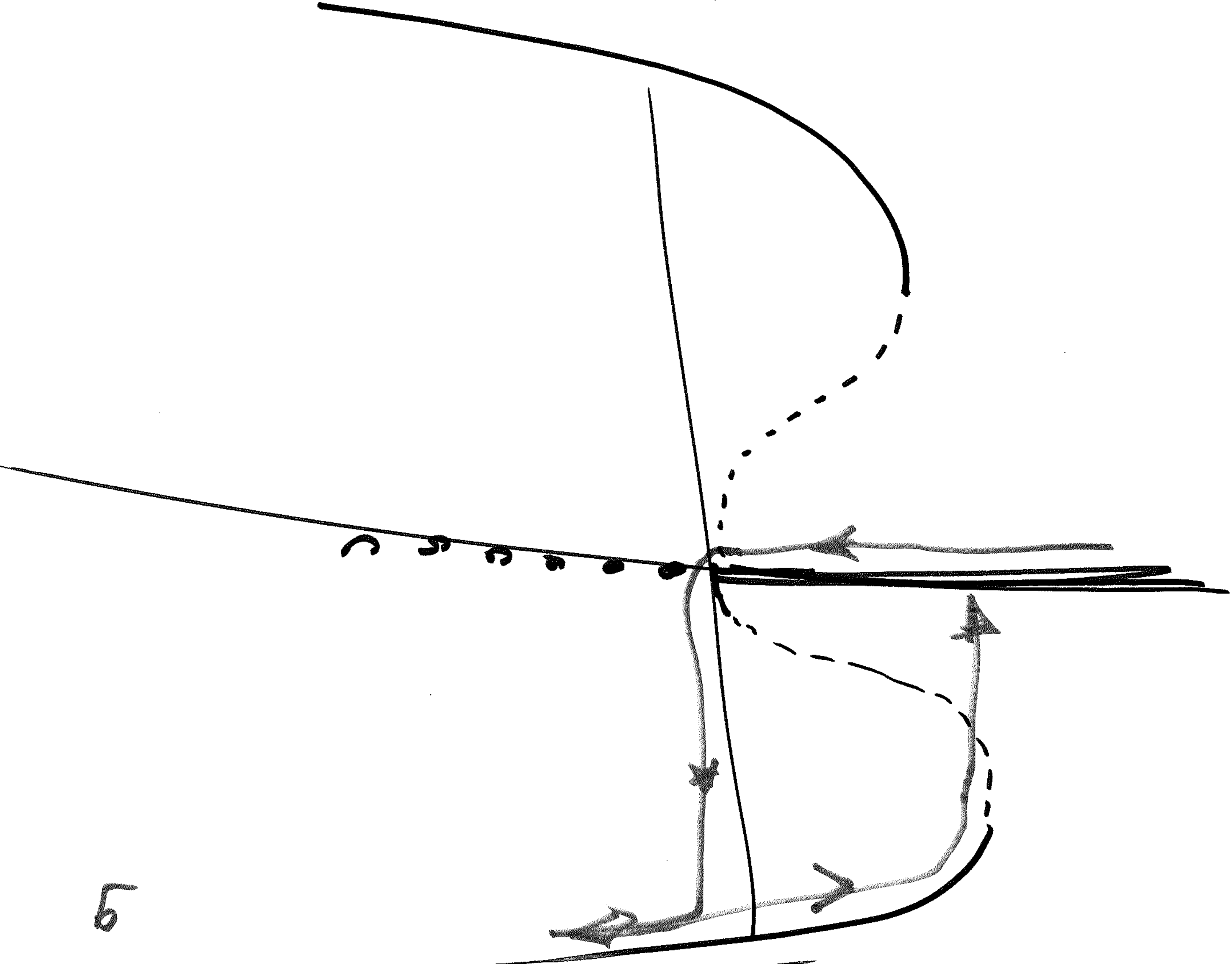
$$= -1 - 4r - \sqrt{1+4r}$$

$$= -(1+4r) - \sqrt{1+4r} < \emptyset, \text{ stable}$$

$x = \emptyset$, $\frac{\partial f}{\partial x} = r$ $\quad r > \emptyset$ unst

$r < \emptyset$ stable


x*
x3
x1
-1/4
r
x2
x4



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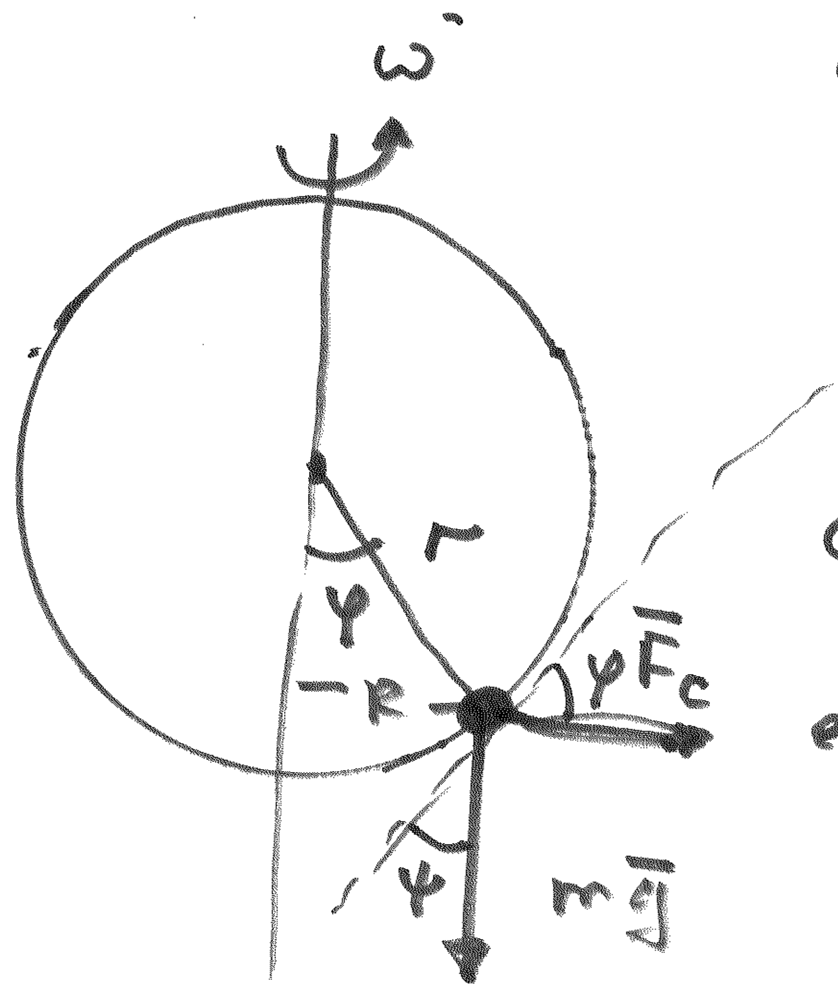


Over damped mass on rotating Hoop

$a = v^2/r = \omega^2 r$ $v = \omega r$

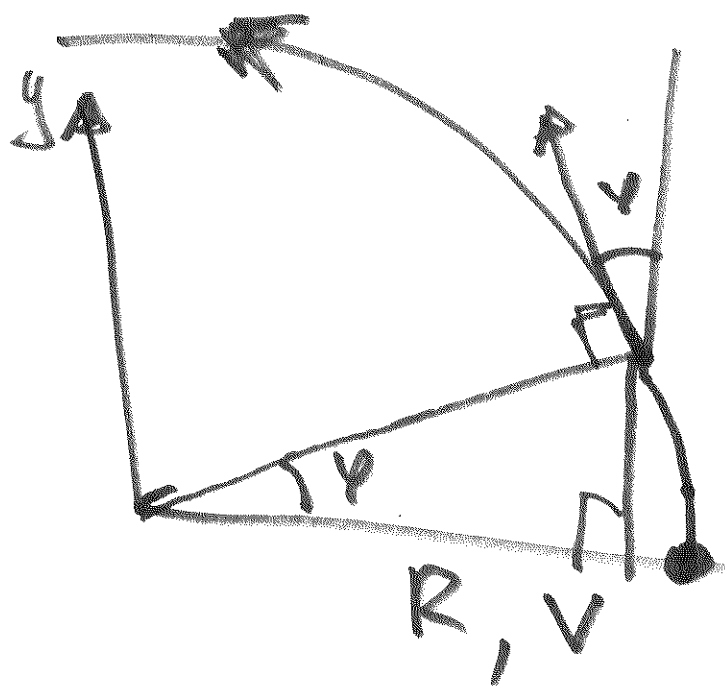
effective radiu

$R = r \sin\psi$

$\bar{F} = m\bar{a}'$

$\Psi = \frac{\pi}{2} - \psi$ $F_c = F_{\text{centrifugal}}$

$= \omega^2 r(\sin\psi) \cdot m$



$t = 0, \quad \bar{x}(t=0) = (R, 0)$

$\bar{V}(t=0) = (0, V)$

$\varphi(dt) = \omega \cdot dt$

$\bar{V}(dt) = V(\cos\varphi, \sin\varphi)$

$= V(\cos(\omega dt), \sin(\omega dt))$

$$\bar{a} = \frac{\bar{V}(dt) - V(t=0)}{dt} =$$

$= V(\cos(\omega dt),$

$v(t=0) = (0, v)$

$\boxed{a = \omega^2 R}$

$v(t=dt) = V(-\sin\varphi, \cos\varphi)$

$= V(-\sin\omega dt, \cos\omega dt)$

$$\bar{a} = \frac{v(dt) - v(0)}{dt} =$$

$$= \frac{V}{dt}\left(-\sin\omega dt, \cos(\omega dt) - 1\right) =$$

$\sin x = x - x^3/3$

$\cos x = 1 - x^2/2$

$$= \frac{V}{dt}(-\omega dt, 0) =$$

$$= -V\omega = -\omega^2 R$$

radius

$$m r \ddot{\varphi} = -\underbrace{\beta \dot{\varphi}}_{\text{friction}} - \underbrace{mg\sin\varphi}_{\text{gravity}} + \underbrace{m\omega^2 r \sin\varphi \cdot \cos\varphi}_{\text{centrifugal}}$$

$$\text{Newton} = 1\,\text{kg} \cdot \frac{1\,\text{meters}}{\text{sec}^2}$$

$$\frac{\text{mass} \cdot \text{length}}{\text{time}^2}$$

$$\frac{\text{mass} \cdot \text{length}}{\text{time}^2}$$

$$[\beta] = \frac{\text{mass} \cdot \text{length}}{\text{time}}$$

$$t = T\,\tau\,; \quad [t] = [T] = \text{time}$$

$$\cancel{[\tau] = 1}$$

$$[\tau] = 1$$

$$\frac{d}{dt} = \frac{1}{T}\frac{d}{d\tau} \qquad \boxed{T = \frac{b}{mg}}$$

$$\frac{mr}{T^2}\frac{d^2}{d\tau^2}\varphi = -\frac{b}{T}\frac{d}{d\tau}\varphi - mg\sin\varphi + m\omega^2 r\sin\varphi\cos\varphi$$

$$\frac{r}{T^2 g}\frac{d^2}{d\tau^2}\varphi = -\overbrace{\frac{b}{T\cdot mg}}^{1}\frac{d}{d\tau}\varphi - \sin\varphi + \frac{\omega^2 r}{g}\sin\varphi\cos\varphi$$

$$\frac{r}{T^2 g} = \frac{r\, m^2 g^2}{g\, b^2} = \frac{r\, m^2 g}{b^2}$$

$$\frac{r\, m^2 g}{b^2} \frac{d^2}{dT^2} \varphi = -\frac{d}{dT}\varphi - \sin\varphi + \frac{\omega^2 r}{g} \sin\varphi \cos\varphi$$

$$\varepsilon = \frac{r\, m^2 g}{b^2}\,; \quad \gamma = \frac{\omega^2 r}{g}\,;$$

$$\varepsilon\, \ddot{\varphi} = -\dot{\varphi} - \sin\varphi + \gamma \sin\varphi \cos\varphi\,; \quad \varphi \equiv \varphi(T)$$

$$\varepsilon \frac{d^2}{dT^2} \varphi(T) = -\frac{d}{dT}\varphi(T) - \sin\varphi + \gamma \sin\varphi \cos\varphi$$

$$\varepsilon \ddot{\varphi} = -\dot{\varphi} - \sin\varphi + \gamma \sin\varphi \cos\varphi$$

$$\varepsilon = \frac{r m^2 g}{b^2}, \text{ very large } b,$$

$$b^2 >> r m^2 g$$

$$\boxed{\gamma > 0}$$

$\varepsilon << 1$, neglect $\ddot{\varphi}$

$$\dot{\varphi} = -\sin\varphi + \gamma \sin\varphi \cos\varphi$$

$$\sin\varphi = 0 \Rightarrow \varphi = 0,\ \varphi = \pi$$

$$\gamma \cos\varphi = 1$$

$$\cos\varphi = 1/\gamma, \quad \varphi = \mathrm{Arc\,Cos}\ 1/\gamma$$

exists if $\gamma > 1$

$$\dot{\varphi} = f(\varphi);$$

$$f(\varphi) = -\sin\varphi + \gamma \sin\varphi \cos\varphi$$

$$\varphi = \emptyset, \quad \varphi = \pi, \quad \varphi = \mathrm{ArcCos}\ 1/\gamma$$