$$\ddot{x} + \epsilon \dot{x} + x = 0$$

$$x = x_0 + \epsilon x_1$$

1. $\ddot{x}_0 + \epsilon \ddot{x}_1 + \epsilon \dot{x}_0 + x_0 + \epsilon x_1 = 0$

2. ϵ^0: $\ddot{x}_0 + x_0 = 0 \Rightarrow x_0(t) = A\cos t + B\sin t$

3. ϵ^1: $\ddot{x}_1 + x_1 = -\dot{x}_0 = \underbrace{+A\sin t - B\cos t}_{\text{resonant forcing}}$

$x_1 \sim t$

$x_1 = Ct\cos t + Dt\sin t$

4. $A \Leftarrow A(\epsilon t)$

5. $B \Leftarrow B(\epsilon t)$

$$x_0(t) = A(\epsilon t)\cos t + B(\epsilon t)\sin t$$

$$\dot{x}_0(t) = \epsilon A'(\epsilon t)\cos t + (-)A(\epsilon t)\sin t$$

$$+ \epsilon B'(\epsilon t)\sin t + B(\epsilon t)\cos t$$

$$= \left(\underline{B(\epsilon t)} + \epsilon A'(\epsilon t)\right)\cos t$$

$$+ \left(\epsilon B'(\epsilon t) - A\underline{(\epsilon t)}\right)\sin t$$

$$\ddot{x}_0(t) = \left(\epsilon B'(\epsilon t)\cos t\right) +$$

$$- \left(B(\epsilon t) + \epsilon A'(\epsilon t)\right)\sin t$$

$$- \epsilon A'(\epsilon t)\sin t$$

$$+ \left(\epsilon B'(\epsilon t) - A(\epsilon t)\right)\cos t$$

ignore ϵ^2 and higher

$$\ddot{x}_0(t) = (2\epsilon B' - A)\cos t + (-B - \epsilon A' \cdot 2)\sin t$$

$$\ddot{x} + \epsilon \dot{x} + x = \emptyset$$

$$+\epsilon \ddot{x}_1 + x_1 + (2\epsilon B' - \underline{A})^{1}\cos t - (\underline{\underline{B}} + 2A'\epsilon)^{2}\sin t$$
$$+ (B\cos t - A^{3}\sin t)\epsilon$$
$$+ \underline{A^{4}\cos t} + \underline{\underline{B^{5}\sin t}} = \emptyset$$

$$\ddot{x}_1 + x_1 + \cos(t)(2B' + \cancel{B}) +$$
$$\sin(t)(2A' - A) = \emptyset$$

$$B' = -B/2 \qquad B(t) = e^{-\frac{\epsilon}{2}t}$$
$$A' = -A/2 \qquad A(t) = e^{-\epsilon t/2}$$

should be $e^{-\epsilon t}$

$$x(t) = x_0(t) + \epsilon x_1(t)$$

$$= A(\epsilon t)\cos t + B(\epsilon t)\sin t + \epsilon x_1$$

$$\ddot{x}_1 + x_1 = \emptyset$$

Van der Paul oscillator

$$\ddot{x} + x + \epsilon(x^2 - 1)\dot{x} = 0$$

$$x = x_0 + \epsilon x_1$$

$$\ddot{x}_0 + x_0 = 0 \Rightarrow x_0 = R\cos(t+\varphi)$$

$$\dot{x}_0 = -R\sin(t+\varphi)$$

$$\underline{\ddot{x}_0} + \epsilon\ddot{x}_1 + \underline{x_0} + \epsilon x_1 = -\underbrace{\epsilon(x_0^2 - 1)\cdot\dot{x}_0}\,;$$

$$\epsilon\ddot{x}_1 + \epsilon x_1 = -\epsilon\left(R^2\cos^2(t+\varphi) - 1\right)(-R)\sin(t+\varphi)$$

$$\cos^2 x \sin x = \frac{1}{4}(\sin x + \sin 3x)$$

$$\ddot{x}_1 + x_1 = +R^3\cos^2(t+\varphi)\sin(t+\varphi) - R\sin(t+\varphi)$$

$$= \frac{R^3}{4}\left(\underline{\sin(t+\varphi)} + \sin(3t+3\varphi)\right) - \underline{R\sin(t+\varphi)}$$

$$x_0(t) = R(\epsilon t)\cos(t + \varphi(\epsilon t))$$

$$\dot{x}_0(t) = \epsilon R'(\epsilon t)\cos(t + \varphi(\epsilon t))$$
$$- R(\epsilon t)\sin(t + \varphi(\epsilon t))(1 + \epsilon\varphi'(\epsilon t))$$

$$\ddot{x}_0(t) = -\epsilon R'(\epsilon t)\sin(t + \varphi(\epsilon t))(1 + \epsilon\cancel{\varphi'}(\epsilon t))$$
$$- \epsilon R'(\epsilon t)\sin(t + \varphi(\epsilon t))$$
$$- R(\epsilon t)\cos(t + \varphi(\epsilon t))(1 + 2\epsilon\varphi'(\epsilon t))$$

$$= -2\epsilon R'(\epsilon t)\sin(t + \varphi(\epsilon t))$$
$$- R(\epsilon t)\cos(t + \varphi(\epsilon t)) - 2\epsilon\varphi'(\epsilon t)\cos(t + \varphi(\epsilon t))$$

$$x = x_0 + \epsilon x_1$$

$$= R(\epsilon t)\cos(t + \varphi(\epsilon t)) + \epsilon X_1$$

$R(\epsilon t) \equiv R$

$\varphi(\epsilon t) \equiv \varphi$

$$\ddot{x} + x + \epsilon(x^2 - 1)\dot{x} = \emptyset$$

$$-2\epsilon R' \sin - \underline{R \cos} - 2\epsilon \varphi' \cos R + \epsilon \ddot{X}_1 \quad | \ddot{x}$$

$$+ \underline{R \cos} + \epsilon X_1 \quad | x$$

$$+ \epsilon (R^2 \cos^2 - 1)(-R \sin) = \emptyset \quad | \epsilon(x^2 - 1)\dot{x}$$

$$\underline{-2R' \sin} - \underline{\underline{2R\varphi' \cos}} + \ddot{X}_1 + X_1 +$$

$$+ \frac{(-R^3)}{4}(\underline{\sin} + \sin 3) + R \underline{\sin} = \emptyset$$

Require no secular terms

$$\sin(t+\varphi)\left(-2R' - \frac{R^3}{4} + R\right) + (-)2R\varphi' \cos(t+\varphi) = \emptyset$$

$$\sin^3 x = (e^{ix} -$$

$$\cos^3 x = (e^{ix} + e^{-ix})^3 / 8$$

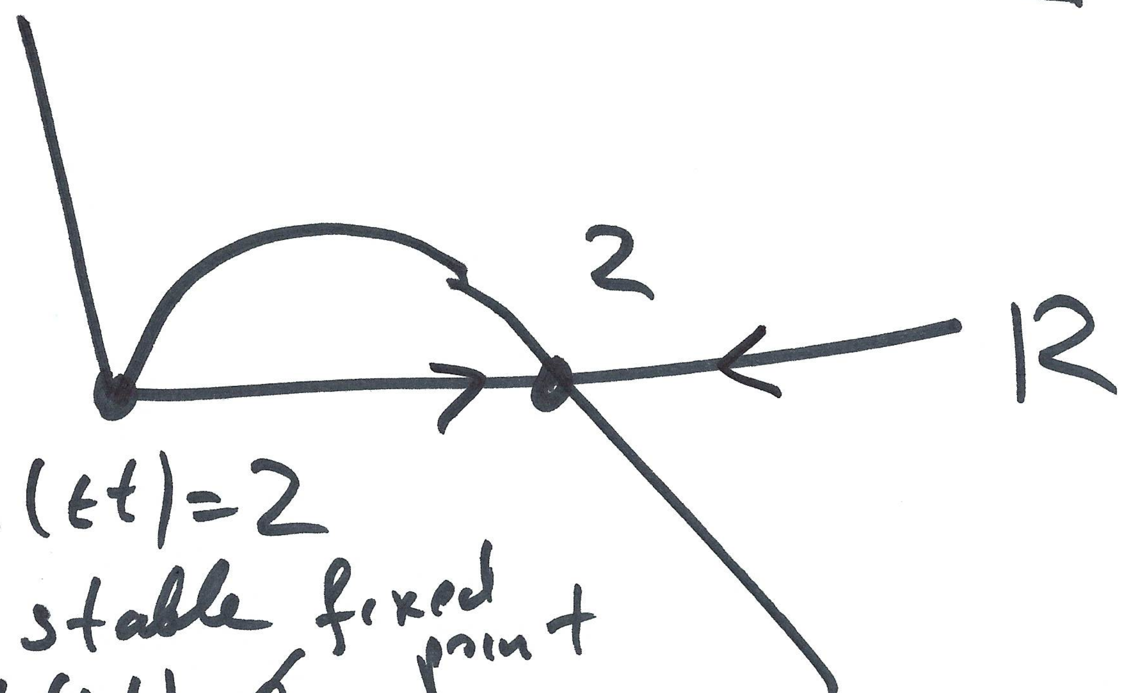
$$= \frac{1}{8}(e^{3ix} + e^{-3ix} + 3e^{ix} + 3e^{-ix})$$

$$-2R' - \frac{R^3}{4} + R = \emptyset$$

$$R\varphi' = \emptyset$$

$$R' = \frac{R}{2}\left(-\frac{R^2}{4} + 1\right)$$

$$\cancel{R} \quad R'(\epsilon t) = -\frac{R^3(\epsilon t)}{8} + \frac{R(\epsilon t)}{2}$$



$R(\epsilon t) = 2$
stable fixed point
$\varphi'(\epsilon t) = \emptyset$

$$\ddot{x}_1 + x_1 = \frac{R^3}{4} \sin(3t + 3\varphi)$$

$$x_1 = A \sin(3t + 3\varphi)$$

$$\ddot{x}_1 = -9A \sin(3t + 3\varphi)$$

$$-9A + A = \frac{R^3}{4}$$

$$A = -\frac{R^3}{32}$$

$$\lim_{t \to \infty} x_0(t) = 2 \sin(t + \varphi)$$