$$(x+a)^2 = x^2 + 2ax + a^2$$

$$y''(x) + 2a\,y'(x) + a^2 y(x) = \emptyset$$

$$\left(\frac{d^2}{dx^2} + 2a\frac{d}{dx} + a^2\right) y(x) = \emptyset$$

$$\left(\frac{d}{dx} + a\right)^2 y(x) = \emptyset$$

$$\left(\frac{d}{dx} + a\right)\underbrace{\boxed{\left(\frac{d}{dx} + a\right) y(x)}}_{z(x)} = \emptyset$$

(1)

$$\frac{d}{dx} z(x) + a\,z(x) = \emptyset$$

$$\frac{dz}{z} = -\,a\,dx$$

$$\frac{dz}{dx} = -a z(x)$$

$$\int \frac{dz}{z} = -\int a\,dx \; ; \quad \ln z = -ax + C$$

$$z(x) = z_0 e^{-ax} \; ; \quad z_0 = z(x=0)$$

② $$\left[\frac{dy(x)}{dx} + a y(x) = z_0 e^{-ax} \right] e^{ax}$$

$$e^{ax} \frac{dy(x)}{dx} + a y(x) e^{ax} = z_0$$

$$\frac{d}{dx}\left(y(x) e^{ax} \right) = z_0$$

$$y(x)e^{ax} = z_0 x + C$$

$$\boxed{y(x) = z_0 x e^{-ax} + C e^{-ax}}$$

$$y_1(x) = x e^{-ax}$$

$$y_2(x) = e^{-ax}$$

$$W(x) = \cancel{|y_1 \, fx} \quad \mathrm{Det}\begin{bmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{bmatrix}$$

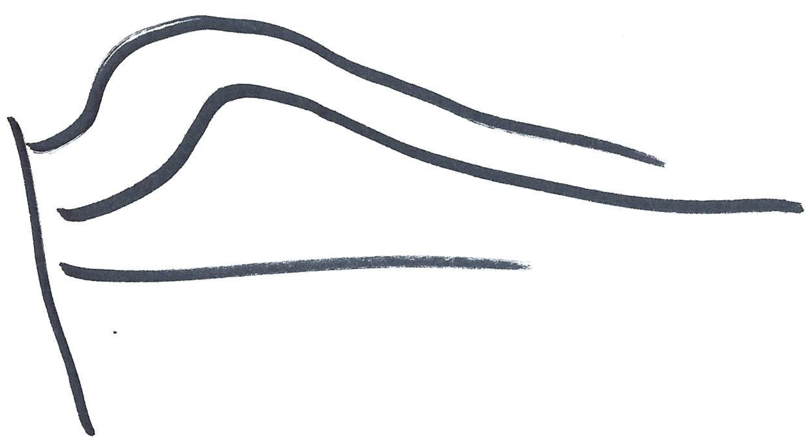
$$= \mathrm{Det}\cancel{\begin{bmatrix} x e^{-ax} \\ e^{-ax}(1-ax) \end{bmatrix}}$$

$$\dot{W}(x) = \mathrm{Det}\begin{bmatrix} x e^{-ax} & e^{-ax} \\ e^{-ax}(1-ax) & -ae^{-ax} \end{bmatrix}$$

$$= -ax\, e^{-2ax} - e^{-2ax} + ax\, e^{-2ax}$$

$$= -e^{-2ax} \neq \emptyset$$

$a > 0$



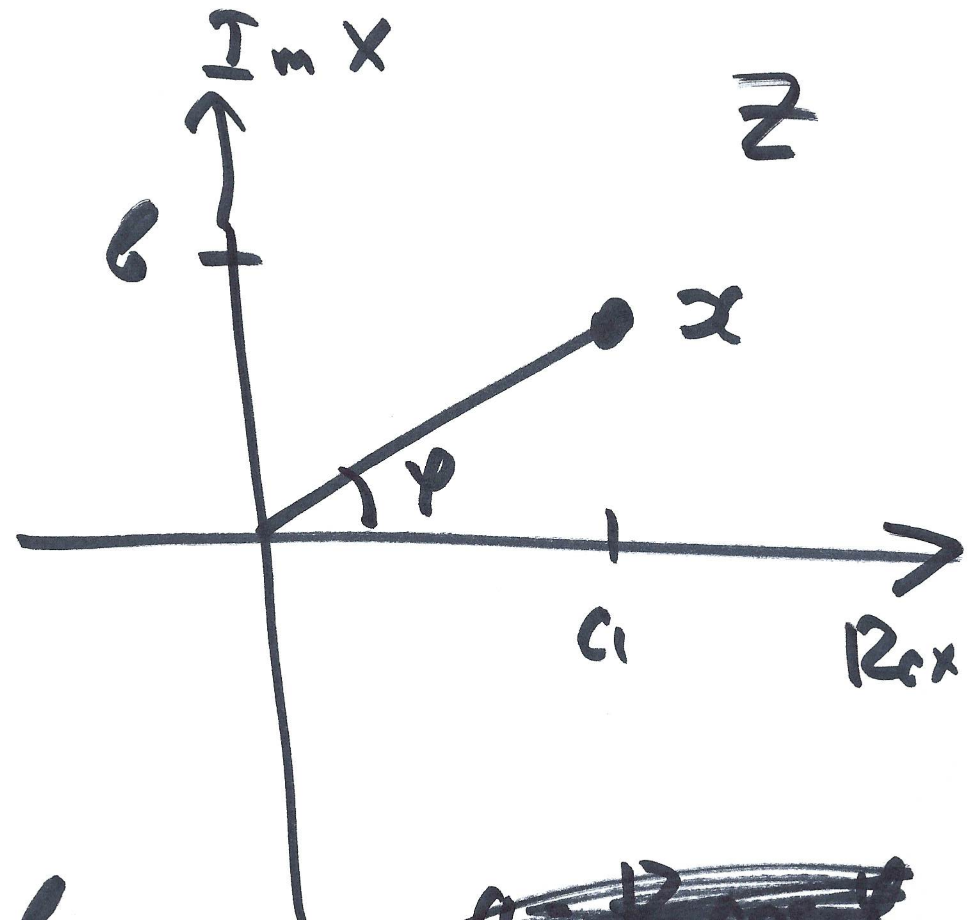
$i = \sqrt{-1}$

$$\boxed{x = r e^{i\varphi}}$$

$x = a + ib$

$a = \mathrm{Re}(x)$

$b = \mathrm{Im}(x)$



Conjugate

$x^* = \bar{x} = a - ib$

$r = |x| = \sqrt{a^2 + b^2}$

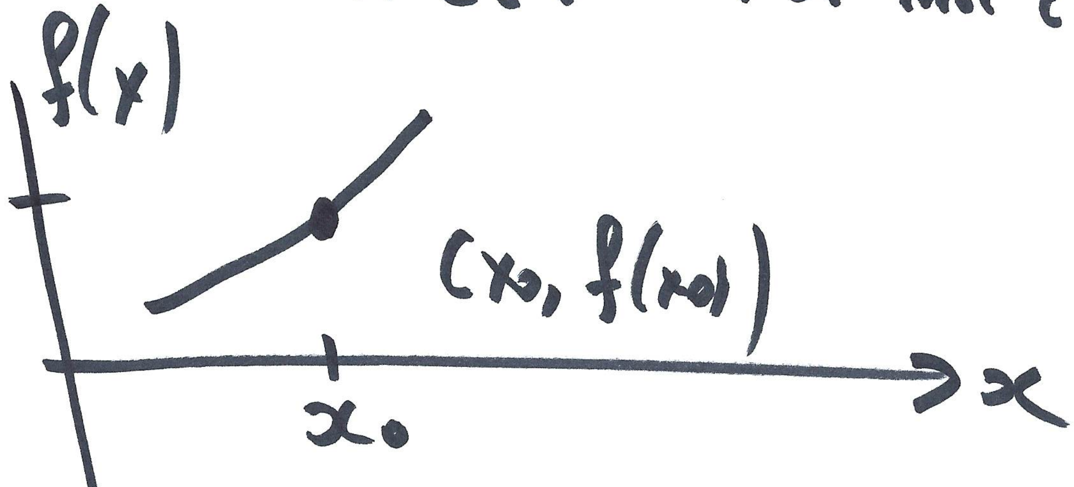
$\tan\varphi = b/a$; $\varphi = \arctan(b/a)$

$\cancel{a = r\cos\varphi}$

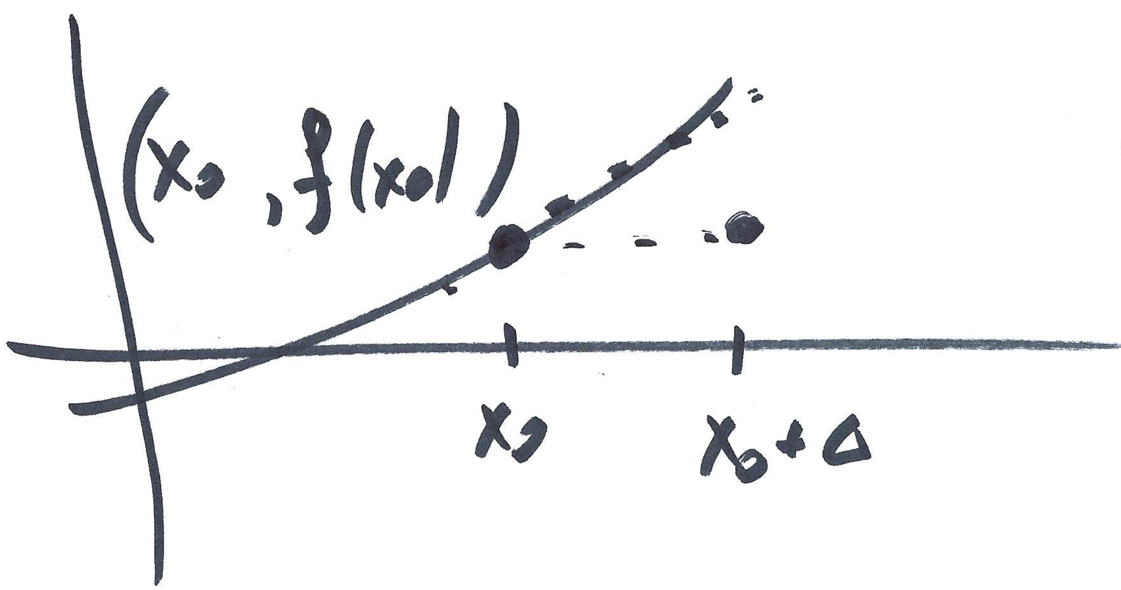
$a = r\cos\varphi$

$b = r\sin\varphi$

Euler Formula



$$f(x_0+\Delta) = f(x_0) + f'(x_0)\Delta + f''(x_0)\Delta^2/2 + f'''(x_0)\Delta^3/6 + \ldots$$



$$f(x+\Delta) = \sum_{k=\emptyset}^{\infty} \left(\frac{d}{dx}\right)^k f(x) \cdot \frac{\Delta^k}{k!}$$

$$k! = k \cdot (k-1) \cdot (k-2) \cdots 3 \cdot 2 \cdot 1$$

$$\emptyset! = 1$$

$$(e^x)' = \frac{d}{dx}(e^x) = e^x$$

$$e^x = \sum_{k=\emptyset}^{\infty} \left[\left(\frac{d}{dx}\right)^k e^x\right]_{x=\emptyset} \frac{x^k}{k!} = \sum_{k=\emptyset}^{\infty} \frac{x^k}{k!}$$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$(\sin x)' = \cos(x)$$

$$(\cos x)' = -\sin x$$

$$e^{ix} = \sum_{k=0}^{\infty} \frac{(ix)^k}{k!} = \sum_{k=0}^{\infty} \frac{(ix)^{2k}}{(2k)!} + \sum_{k=0}^{\infty} \frac{(ix)^{2k+1}}{(2k+1)!}$$

$(i)^k$

$(i)^0 = 1$

$(i)^1 = i$

$(i)^2 = -1$

$(i)^3 = -i$

$(i)^4 = -1 \cdot -1 = 1$

$(i)^{2n} = (-1)^n$

$(i)^{2n+1} = i(-1)^n$

!

$$e^{ix} = \boxed{\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}} + \sum \frac{i(-1)^k x^{2k+1}}{(2k+1)!} =$$

sin x

cos x

$$\boxed{e^{ix} = \cos x + i \sin x}$$

Euler Formula

$$e^{ix} = \cos x + i \sin x$$

$$e^{-ix} = \cos x - i \sin x$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\cancel{\sin x = e^{ix} - e^{-i}}$$

$$\sin x = \frac{e^{+ix} - e^{-ix}}{2i}$$

$\sin x \cdot \cos x =$

$$= \frac{e^{ix} - e^{-ix}}{2i} \cdot \frac{e^{ix} + e^{-ix}}{2}$$

$$\cancel{= \frac{e^{2ix} - e^{-}}{2i \cdot 2}}$$

$$= \frac{1}{2 \cdot 2i} \overbrace{\left(e^{2ix} - e^{-2ix} \right)}^{\sin 2x}$$

$$= \frac{1}{2} \sin 2x$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos(x+y) = \frac{1}{2}\left(e^{i(x+y)} + e^{-i(x+y)}\right)$$

$$\cos x \cos y =$$

$$= \frac{1}{4}\left(e^{ix} + e^{-ix}\right)\left(e^{iy} + e^{-iy}\right)$$

$$= \boxed{\frac{1}{4}\left(e^{i(x+y)} + e^{-i(x+y)}\right.} + \boxed{\left.e^{i(x-y)} + e^{-i(x-y)}\right)}$$

$$\sin x \sin y = \frac{1}{2i \cdot 2i}\left(e^{ix} - e^{-ix}\right)\left(e^{iy} - e^{-iy}\right)$$

$$= -\frac{1}{4}\left(\boxed{e^{i(x+y)} + e^{-i(x+y)}} - \boxed{e^{i(x-y)} - e^{-i(x-y)}}\right)$$

$$\boxed{\cos(x+y) = \cos x \cos y - \sin x \sin y}$$

$$y''(x) - 2y'(x) + 2y(x) = \emptyset$$

$$y(x) = e^{rx};$$

$$r^2 - 2r + 2 = \emptyset;$$

$$r_{12} = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2}$$

$$= 1 \pm i$$

$$y(x) = C_1 e^{(1+i)x} + C_2 e^{(1-i)x}$$

$$= C_1 e^{x} e^{ix} + C_2 e^{x} e^{-ix}$$

$$= e^{x}(C_1 e^{ix} + C_2 e^{-ix})$$

$$y(x) = e^{x}\left(C_1 \cos x + C_1 i \sin x\right)$$

$$+ e^{x}\left(C_2 \cos x - i C_2 \sin x\right)$$

$$= e^{x} \overbrace{\boxed{\left(C_1 + C_2\right)}}^{D_1} \cos x$$

$$+ e^{x} \overbrace{\boxed{i\left(C_1 - C_2\right)}}^{D_2} \sin x$$

$$= D_1 e^{x} \cos x + D_2 e^{x} \sin x$$

$$c_1 = a + i\,b$$

$$c_2 = c + i\,d$$

$$c_1 + c_2 = (a + c) + i\,(b + d)$$

$$i\,(c_1 - c_2) = i(a - c)\ (b - d)$$

$$a y''(x) + b y'(x) + c y(x) = 0;$$

$$b^2 < 4ac$$

$\gamma = \mathrm{Re}\, r$

Characteristic equation $\omega = \mathrm{Im}$

$\omega = \mathrm{Im}\, r$

$$a r^2 + b r + c = 0$$

$$r_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm i \sqrt{4ac - b^2}}{2a} =$$

$$= \gamma + i\,\omega; \quad \gamma = -b/2a$$

$$\omega = \sqrt{4ac - b^2}/(2a)$$

$$y(x) = C_1 e^{r_1 x} + C_2 e^{rx} \qquad r_1 = \gamma + i\omega \quad r_2 = \gamma - i\omega$$

$$= C_1 e^{\gamma x} e^{i\omega x} + C_2 e^{\gamma x} e^{-i\omega x}$$

$$= C_1 e^{\gamma x}(\cos \omega x + i \sin \omega x)$$

$$+ C_2 e^{\gamma x}(\cos \omega x - i \sin \omega x) \qquad \boxed{e^{ix} = \cos x + i \sin}$$

$$= (C_1 + C_2) e^{\gamma x} \cos \omega x$$

$$+ i (C_1 - C_2) e^{\gamma x} \sin \omega x$$

$$= D_1 e^{\gamma x} \cos(x \cdot \omega) + D_2 e^{\gamma x} \sin(\omega x)$$

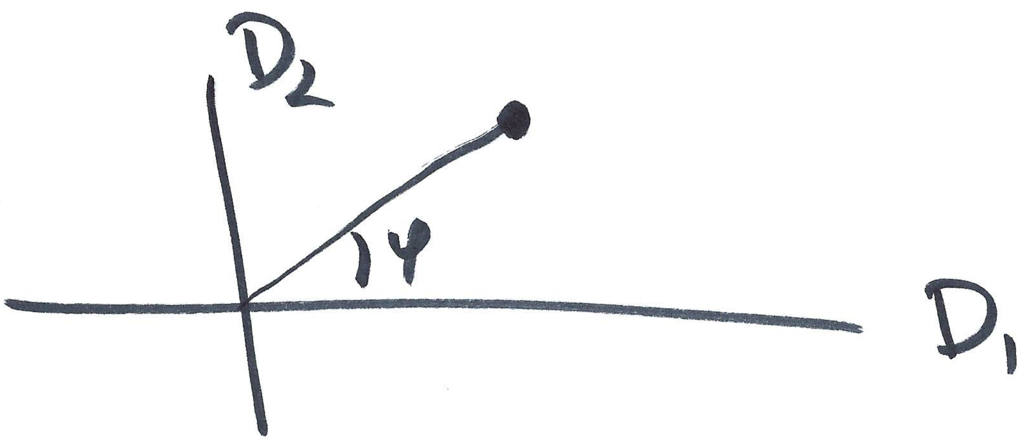
$$a y''(x) + b y'(x) + c y(x) = 0$$

$$\cancel{r_{1,2} = b^2}$$

$$r_{1,2} = \frac{-b \pm i\sqrt{4ac - b^2}}{2a}$$

$$y(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x}$$

$$y(x) = D_1 e^{(\mathrm{Re}\, r_1)x} \cos(\mathrm{Im}(r_1) x) + D_2 e^{(\mathrm{Re}\, r_1)x} \sin(\mathrm{Im}(r_1) x)$$



$$D_1 = R\cos\varphi$$

$$D_2 = R\sin\varphi$$