$$v(t) = \frac{dx(t)}{dt}$$

$$x(t) = \int_0^t v(t)\,dt \qquad C = 1160 \text{ meters}$$

$$v(t) = 40\,\frac{\text{meters}}{\text{sec}}\left(-1 + e^{-\frac{t}{4\text{sec}}}\right)$$

$$x(t) = \int_0^t 40\left(-1 + e^{-t/4\text{sec}}\right)dt \cdot \text{meters} + C$$

$$= -40t - \frac{40e^{-t/4\text{sec}}}{1/(4\text{sec})} \cdot \text{meters} + C$$

$$= -40t - 160e^{-t/4\text{sec}} \cdot \text{meters} + C$$

$$x(t=0) = \underset{\text{meters}}{-160} + C = 1000 \text{ meters}$$

2 13

$$x(t) = -40t - 160\, e^{-\frac{t}{4\,sec}} \cdot \text{meters} + 1160 \text{ meters}$$

Question 3 - what is terminal velocity

Way 1: $\lim_{t \to \infty} v(t) =$

$$= \lim_{t \to \infty} \left(40 \text{ meters/sec} \left(-1 + e^{-t/4\,sec} \right) \right)$$

$$= 40 \text{ meters/sec}$$

2 way

$$C = \frac{1}{2} \frac{kg}{sec}$$

$$c\,v = m g$$

$$v = \frac{mg}{c} = \frac{2\,kg \cdot 10\, m/sec^2}{1/2\, kg/sec} = 40 \frac{m}{sec}$$

Question 4

when does the Ball hit the ground?

$$x(t) = -40t \underbrace{- 160e^{-t/4\,sec}}_{\text{neglecting}} + 1160 \text{ meters} = \emptyset$$

$$t = \frac{1160}{40} \text{ sec} = 29 \text{ seconds}$$

$$t = 28.997$$

- Population growth
- Radioactive decay
- mixing
- Newton and nonlinear cooling
- 2nd Newton law

$P = 80$ people

Gossip:

$\cancel{P(t)}$

$$\frac{d}{dt} N(t) = K\, N(t)\,(80 - N(t))$$

Initially Number of peoples doubles every 3 days

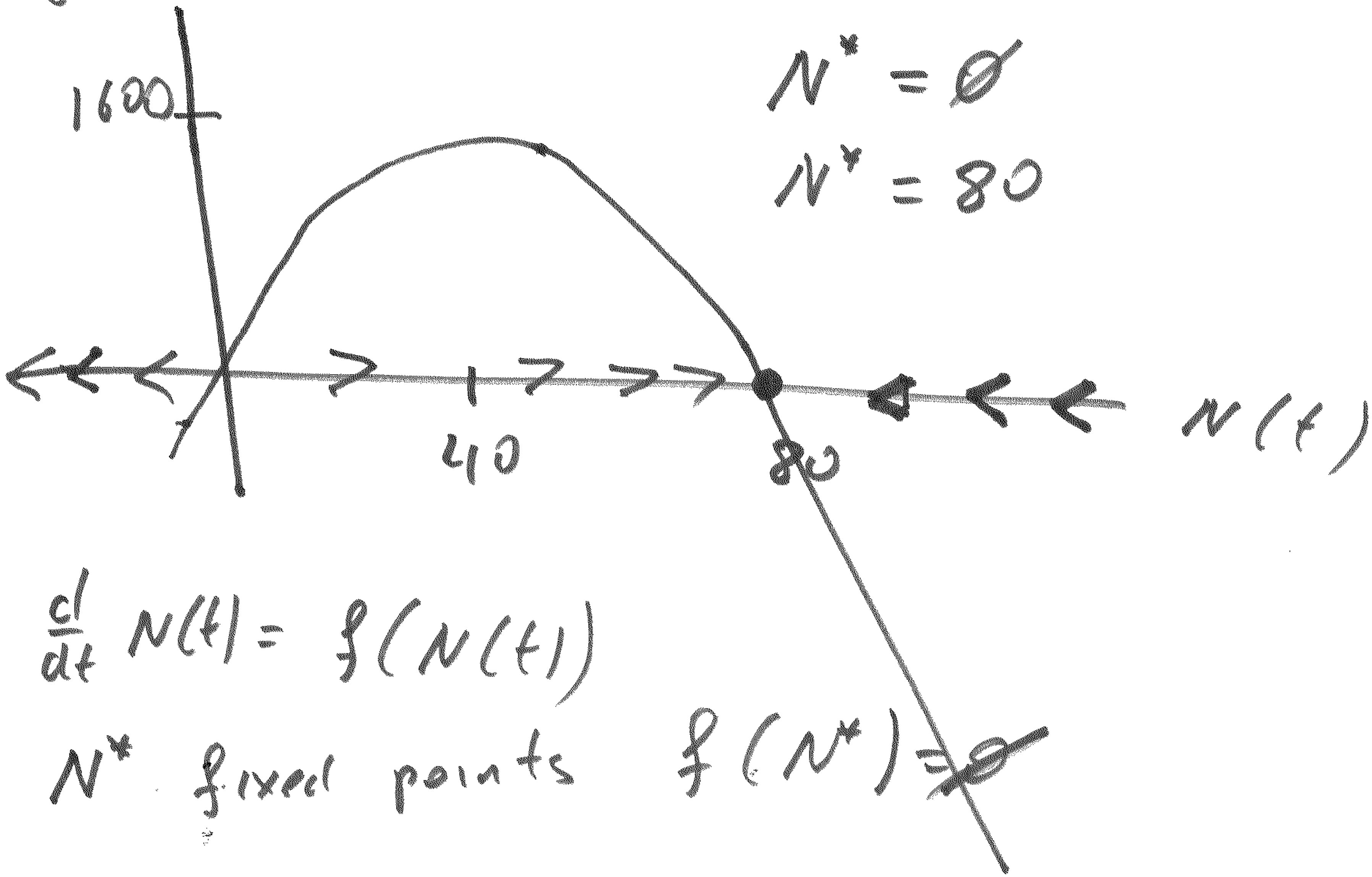
$$\frac{d}{dt} N = K \cdot N \cdot 80$$

$$N(t) = N_0 e^{80K \cdot t}$$

$$2N_0 = N_0 e^{80K \cdot 3\,\text{days}} \; ; \; K = \frac{\ln 2}{240 \text{ days}}$$

$$\frac{d}{dt} P(t) = In - Out$$

$$\frac{d}{dt} N(t) = N(t)(80 - N(t))$$



$N^* = 0$

$N^* = 80$

$$\frac{d}{dt} N(t) = f(N(t))$$

N^* fixed points $f(N^*) = 0$

Max of $N(80-N) = f(N)$

$f'(N) = 80 - 2N = \emptyset \Rightarrow N = 40$

$f(40) = 40(40) = 1600$

① Choose Initial condition $N(t=\emptyset) = \emptyset$

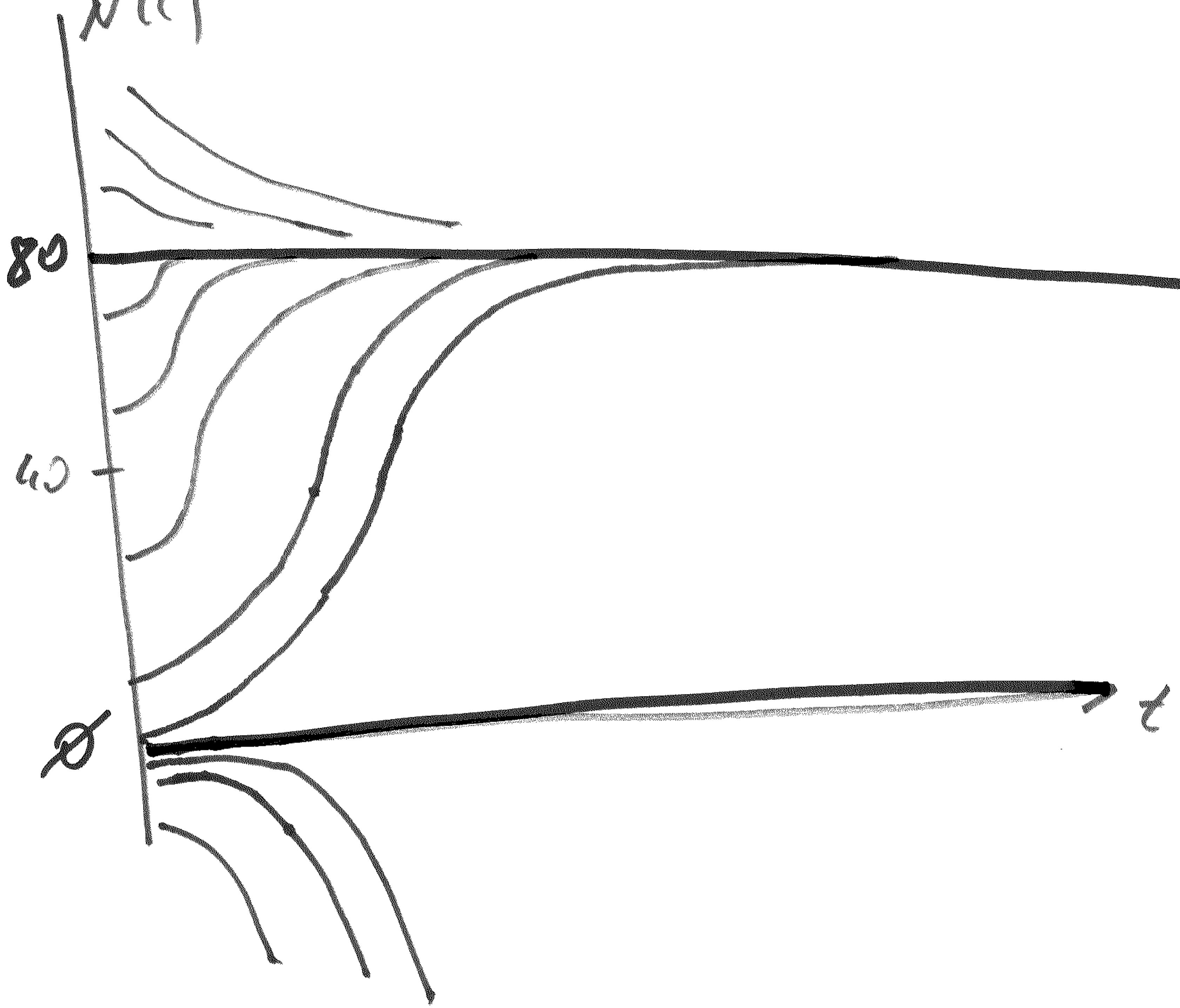
then $N(t) = \emptyset \ \forall t$

$\hookrightarrow$ for all

② $N(t=\emptyset) = 80 \Rightarrow N(t) = 80 \ \forall t$

③ $N(t=\emptyset) = 100, \ \frac{dN(t)}{dt} < \emptyset$

$N(t)$ decrease


N(t)
80
40
Ø
t

$N(t=\emptyset)=5$

Stable fixed point

$$\frac{d}{dt}x(t) = f(x(t));$$

if x^* is fixed point $f(x^*)=\emptyset$

if $x(t=\emptyset) = x^* + \epsilon, |\epsilon| \ll 1$

then $\lim_{t\to\infty} x(t) = x^*$

Unstable fixed point — " — $\lim_{t\to\infty} x(t) \neq x^*$

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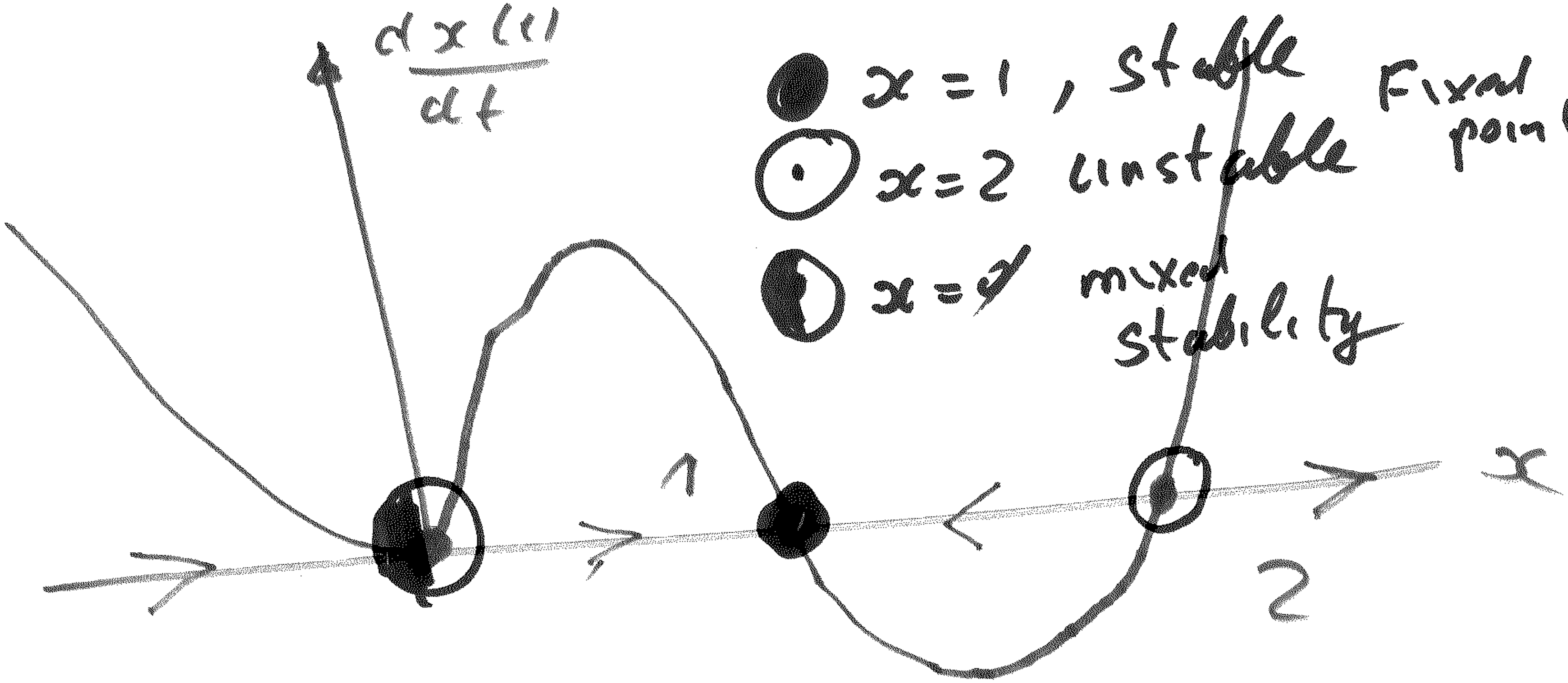
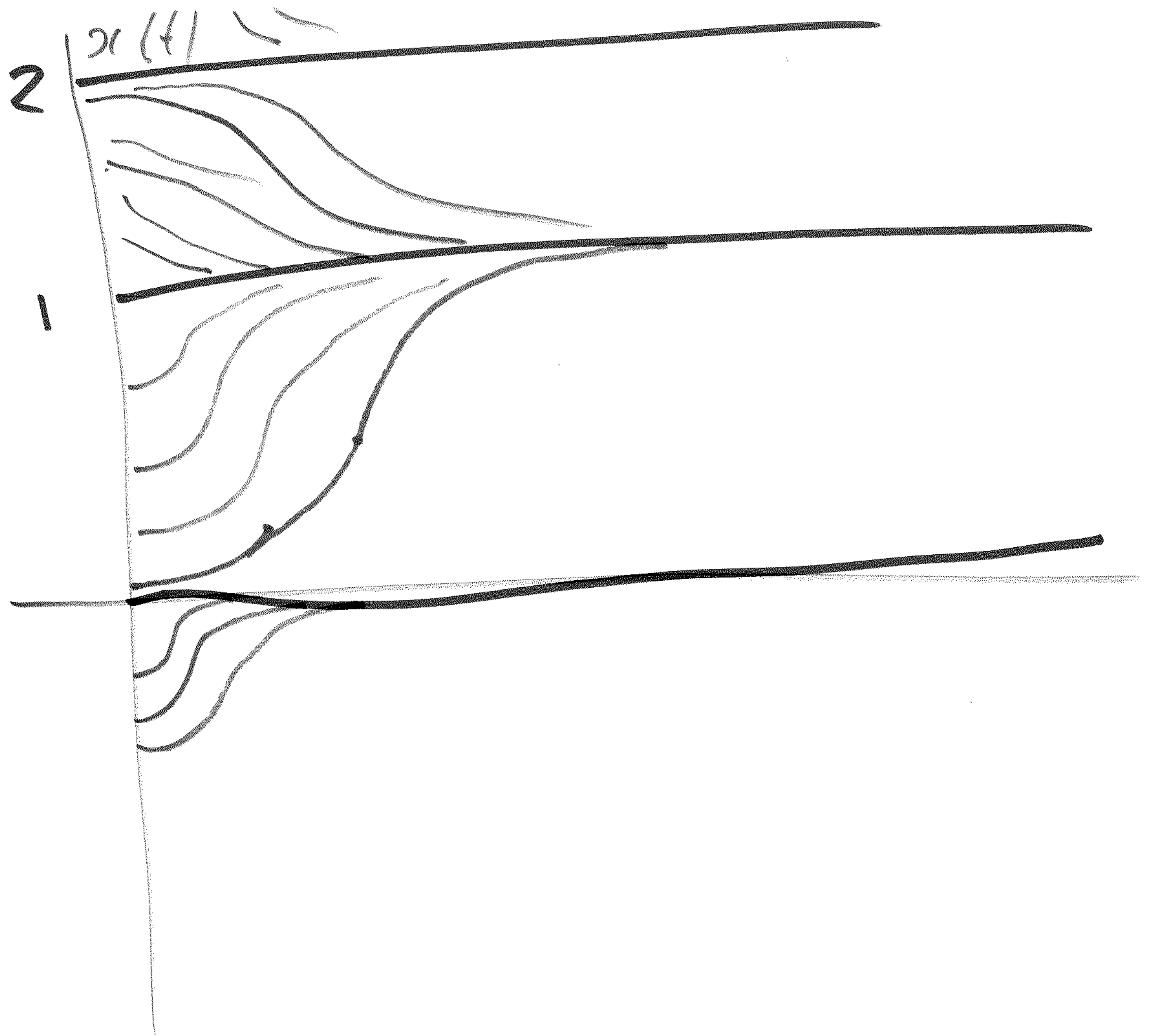
$$\frac{d}{dt}x(t) = x^2(t)(1-x(t))(2-x(t))$$

Find fixed points:

$$\frac{d}{dt}x(t) = f(x(t)), \quad \text{find } x^* \text{ s.t. } f(x^*) = 0:$$

$$x^* = 0; \quad x^* = 1, \quad x^* = 2$$




x(t)
2
1
t
10

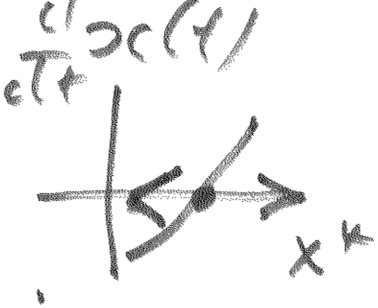
To solve $\frac{d}{dt} x(t) = f(x(t))$

① Find

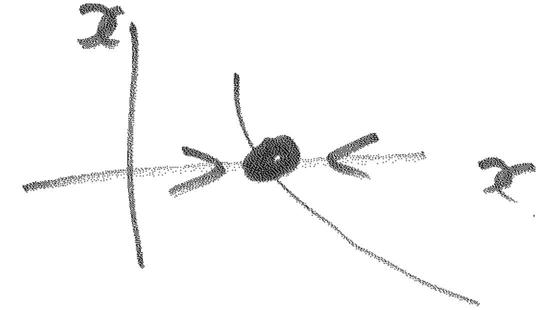
x^* such that $f(x^*) = 0$.

x^* is a fixed point

② If $f(x^*)$ is increasing:
then x^* is unstable fixed point



③ If $f(x^*)$ is decreasing:
then x^* is stable



④ Between fixed points
If $f(x) > 0$, then $\frac{d}{dt} x(t) > 0$
increases

⑤ -//- $f(x) < 0$, -//- $x(t)$ decreases

- Separation of variables

$$\frac{d}{dt} x(t) = f(x(t))\, g(t),$$

$$\int \frac{dx(t)}{f(x(t))} = \int g(t)\,dt$$

- Integrating factor

$$\frac{dx(t)}{dt} + f(t)\, x(t) = g(t),$$

$$\mu(t) = e^{\int f(t)\,dt}$$

$$\frac{d}{dt}\left(x(t) \cdot \mu(t) \right) = g(t)\,\mu(t)$$

- Modeling
- elements of phase plane

Second order ordinary differential equations.

General second order ODE:

$$\frac{d^2}{dx^2} y(x) = F\left(x,\ y(x),\ \frac{dy(x)}{dx} \right)$$

$$y(x = x_0) = y_0$$

$$\frac{dy(x = x_0)}{dx} = y_0'$$

$F = ma$

$$mg = m \frac{d^2 x(t)}{dt^2}$$

$$\ddot{x}(t) \equiv \frac{d^2 x}{dt^2} = g$$

Linear Second order ordinary differential equations.

$$\frac{d^2}{dx^2} y(x) + P(x) \frac{d}{dx} y(x) + Q(x) y(x) = F(x);$$

General homogeneous linear second order ODE : $F(x) = \emptyset$

$$\frac{d^2}{dx^2} y(x) + P(x) \frac{d}{dx} y(x) + Q(x) y(x) = \emptyset \quad (*)$$

Let $y_1(x)$ and $y_2(x)$ that are solutions of $(*)$

$$y(x) = C_1 \, y_1(x) + C_2 \, y_2(x)$$

$y(x)$ is a Linear combination of y_1 and y_2.

$y(x)$ is also a solution of $(*)$