$$m\ddot{x} + 2\gamma\dot{x} + \kappa x = F\cos\omega t$$

$F = -\kappa x$; Lvov's research: $F = -\kappa x - \alpha x^2 - \beta x^3$

$\frac{1}{m}$: $$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f\cos\omega t;$$

- Free oscillator $\beta = f = 0$;

  4 version's of solution

  $$x(t) = R\cos(\omega_0 t - \varphi)$$

- Damped oscillator
  - overdamped
  - critically damped
  - underdamped

Forced oscillator : $\beta = 0,\ f \neq 0$

nonresonant forcing $\omega_0 \neq \omega$

$$x(t) = \underbrace{R\cos(\omega_0 t - \varphi)}_{\text{free oscillation}} + \underbrace{\underbrace{\frac{f}{\omega_0^2 - \omega^2}}_{?}\cos\omega t}_{\text{forced oscillation}}$$

Resonant forcing $\omega = \omega_0$

$$\ddot{x} + \omega_0^2 x = f \cos\omega_0 t;$$

$$x_p(t) = A \cos\omega_0 t + B \sin\omega_0 t;$$

– a solution to homogeneous equation

$$x_p(t) = A\underline{t} \cos\omega_0 t + B\underline{t} \sin\omega_0 t;$$

$$\dot{x}_p(t) = A\cos\omega_0 t + B\sin\omega_0 t - At\omega_0 \sin\omega_0 t + Bt\cos\omega_0 t \cdot \omega_0$$

$$\ddot{x}_p(t) = -2\omega_0 A \sin\omega_0 t + 2B\omega_0 \cos\omega_0 t - At\omega_0^2 \cos\omega_0 t - B\omega_0^2 t \sin\omega_0 t$$

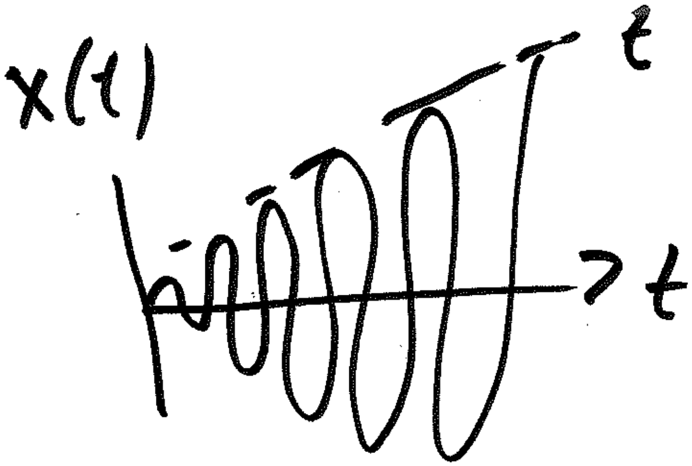
$$-2\omega_0 A \sin\omega_0 t + 2B\omega_0 \cos\omega_0 t$$
$$- A t\, \omega_0^2 \cos\omega_0 t - B\omega_0^2 t \sin\omega_0 t$$
$$+ \omega_0^2 A t \cos\omega_0 t + \omega_0^2 B t \sin\omega_0 t$$

$$= f \cos\omega_0 t$$

$$(\sin\omega_0 t)\left[-2\omega_0 \cancel{- B\omega_0^2} \cdot A\right]$$
$$(\cos\omega_0 t)\left[2B\omega_0 - f\right] = \emptyset$$

$$A = \emptyset, \quad B = f/(\omega_0 \cdot 2)$$

$$x(t) = R\cos(\omega_0 t + \varphi) + \frac{f \cdot t}{2\omega_0}\sin\omega_0 t$$




$F = -kx$ - Becomes invalid

$$F = -kx + \frac{\alpha x^2}{2} - \beta x^3/3 + \ldots$$

Full oscillator:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f\cos\omega t;$$

Homogeneous: $\ddot{x}_0 + 2\beta\dot{x}_0 + \omega_0^2 x_0 = 0;$

$$\lambda^2 + 2\beta\lambda + \omega_0^2 = 0$$

$$\lambda_{1,2} = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$$

$$= -\beta \pm i\sqrt{\omega_0^2 - \beta^2}$$

$$x_0(t) = R e^{-\beta t} \cos\left(\sqrt{\omega_0^2 - \beta^2}\, t + \varphi\right)$$

2 free parameters: $R, \varphi$

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f\cos\omega t$$

$$x = A\cos\omega t + B\sin\omega t$$

$$\dot{x} = -A\omega\sin\omega t + B\omega\cos\omega t;$$

$$\ddot{x} = -A\omega^2\cos\omega t - B\omega^2\sin\omega t;$$

$$\left.\begin{aligned} &-A\omega^2\cos\omega t - B\omega^2\sin\omega t \\ &-2A\beta\omega\sin\omega t + 2\beta B\omega\cos\omega t \\ &+A\omega_0^2\cos\omega t + B\omega_0^2\sin\omega t \end{aligned}\right]\begin{aligned} &\ddot{x} \\ &+2\beta\dot{x} \\ & \end{aligned}$$

$$= f\cos\omega t$$

$$(\cos\omega t)\left(-A\omega^2 + 2\beta B\omega + A\omega_0^2 - f\right) + (\sin\omega t)\left(-B\omega^2 - 2A\beta\omega + B\omega_0^2\right) = \emptyset$$

$$(\,) = \emptyset \Rightarrow \quad 2A\beta\omega = B(\omega_0^2 - \omega^2)$$

$$A = \frac{B}{2\beta\omega}\left(\omega_0^2 - \omega^2\right)$$

$$A\left(-\omega^2 + \omega_0^2\right) + 2\beta B\omega = f$$

$$-\frac{B\left(\omega_0^2 - \omega^2\right)^2}{2\beta\omega} + 2\beta\omega B = f$$

$$B\left(2\beta\omega - \frac{\omega_0^2 - \omega^2}{2\beta\omega}\right) = f$$

$$B = \frac{f}{2\beta\omega - \frac{\left(\omega_0^2 - \omega^2\right)^2}{2\beta\omega}} = \frac{2f\beta\omega}{4\beta^2\omega^2 + \left(\omega^2 - \omega_0^2\right)^2}$$

$$A = \frac{f\left(\omega_0^2 - \omega^2\right)}{4\beta^2\omega^2 + \left(\omega^2 - \omega_0^2\right)^2}$$

$$x = A\cos\omega t + B\sin\omega t$$

$$= R \cdot \cos(\omega t + \varphi)$$

$$R = \sqrt{A^2 + B^2}\,; \quad \tan\varphi = B/A$$

$$R = \frac{f}{\sqrt{m^2(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}}$$

$$\tan\varphi = \ldots$$

$$A = \frac{f(\omega_0^2 - \omega^2)}{4\beta^2\omega^2 + (\omega^2 - \omega_0^2)^2}$$

$$B = \frac{2f\beta\omega}{4\beta^2\omega^2 + (\omega^2 - \omega_0^2)^2}$$

$$R = \sqrt{A^2 + B^2} = \frac{f}{4\beta^2\omega^2 + (\omega^2 - \omega_0^2)^2} \cdot \sqrt{f^2(\omega_0^2 - \omega^2)^2 + 4f^2\beta^2\omega^2}$$

$$= \frac{f}{\sqrt{\cancel{f^2}(\omega_0^2 - \omega^2)^2 + 4\cancel{f^2}\beta^2\omega^2}}$$

$$x(t) = R e^{-\beta t} \cos\left(\sqrt{\omega_0^2 - \beta^2}\, t - \varphi\right) + \frac{f \cos(\omega t - \psi)}{\sqrt{\cancel{f}^2(\omega_0^2 - \omega^2)^2 + 4\beta^2 f \omega^2}}$$

$$\tan \varphi = \frac{2\beta\omega}{\omega_0^2 - \omega^2}$$

Method of complex amplitude

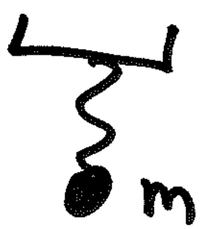
mass. 5 kg, $\Delta x = 10$ centimeters

2 Newtons 4 cm/sec

$F = 10 \sin\left(\frac{t}{2\sec}\right)$ Newton

$v_0 = 3$ cm/sec, $x_0 = \emptyset$

$m\ddot{x} + \gamma\dot{x} + kx = F$




$k \cdot \Delta x = mg$

$$k = \frac{mg}{\Delta x} = \frac{5\,\text{kg} \cdot 10\,\frac{\text{m}}{\text{sec}^2}}{10\,\text{cm}} =$$

$$= \frac{5\,\text{kg}\ 10\ \text{meters}}{\text{sec}^2 \cdot 0.1\ \text{meters}} = 500\ \frac{\text{kg}}{\text{sec}^2}$$

$$F = \gamma \dot{x} = 2\ \text{Newtons} = \gamma \cdot 4\ \text{cm/sec}$$

$$\gamma = \frac{2\ \text{Newtons}}{4\ \text{cm/sec}} = \frac{2 \cdot \text{kg} \cdot \text{meters} \cdot \text{sec}}{0.04\ \text{meters}\ \text{sec}^2}$$

$$= \frac{50\ \text{kg}}{\text{sec}}$$

$$5\,\text{kg} \cdot \ddot{x} + \frac{50\ \text{kg}}{\text{sec}}\dot{x} + 500\ \frac{\text{kg}}{\text{sec}^2} \cdot x$$

$$= 10 \sin\left(\frac{t}{2\,\text{sec}}\right) \cdot \text{Newton}$$

$$x(t=0) = 0$$

$$\dot{x}(t=0) = 0.03\ \text{meters/sec}$$

$$[x] = \text{meters};\ [\dot{x}] = \frac{\text{meters}}{\text{sec}} \quad [\ddot{x}] = \frac{\text{meters}}{\text{sec}^2}$$

Change variables

$x = 1\,\text{meter} \cdot y; \quad t = 1\,\text{sec} \cdot \tau$

To get

$$5\,\text{kg} \cdot \frac{1\,\text{meter}}{\text{sec}^2} \frac{d^2 y}{d\tau^2} + \frac{50\,\text{kg meter}}{\text{sec}^2} \cdot \frac{d}{d\tau} y + 500\,\frac{\text{kg}}{\text{sec}^2} \cdot 1\,\text{meter} \cdot y = 10 \sin\left(\frac{\tau}{2}\right) \cdot \text{Newton}$$

$$\text{Newton} = 1\,\text{kg} \cdot 1\,\text{meter} / \text{sec}^2$$

CANCELS

Consider

$$\ddot{u} + 2\dot{u} + 2u = 5\sin t$$

$$\dot{u}(\emptyset) = u(\emptyset) = \emptyset;$$

Find solution,

does $\lim_{t\to\infty} u(t)$ depend upon initial condition?