MATH 2400
INTRODUCTION TO DIFFERENTIAL EQUATIONS

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Tuesday-Friday 10:00-11:20 **West Hall**
Office Hours : Monday 18-20, Webex meeting rcdcoom lvovy
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TA: Jason Dai daiz4@rpi.edu
Drop in Tutoring: Mon/Tues/Wed/Thurs - 8 pm - 10 pm in DCC XXX
Recitations: M2, M3, R2, R3

August 29, 2024

google

Midterms are on October 1, November 1 and December 3, 2024
There will be no lecture on December 6 the make up lecture will be on November 20(?)

If you are late to class, you agree to either dance, tell a joke or sing.

1 Outline

First-order differential equations, second-order linear equations, eigenvalues and eigenvectors of matrices, systems of first-order equations, stability and qualitative properties of nonlinear autonomous systems in the plane, Fourier series, separation of variables for partial differential equations.

Prerequisites: MATH 1020 and some knowledge of matrices.

2 Learning Outcomes:

To learn the basics of differential equations (as described in the outline and recommended problems) which are a crucial tool in higher-level scientific and engineering subjects. An additional benefit will be to improve mathematical manipulation, modeling, and reasoning skills.

Table 1: Outline

Week	Pages	Title	Sections	Suggested Problems
1	13	Intro/First Order DFQ, Separable Eqn	1.1,1.2, 2.1	1.2.3-4 2.1.1-2 2.1.6
2	24	Integrating Factor and Modeling	2.2, 2.3	2.2.1-3 2.3.2-3 2.3.8
3	40	Steady State and Stability	2.4	2.4.2-6, 2.4.10
4	48	Second Order Linear Equation	3.1-3.5	3.2.4-5, 3.5.3-4,3.5.6-7
5	60	Inhomogeneous Eqn and MethUndetCoef	3.6-3.8	3.8.1-2, 3.8.4-5
6	76	Variation of a Parameter, Mechanical Vibrations, Euler Eq.	3.9-11	3.9.1-4,3.10.1-12,3.11.1-2
7	98	Linear Systems, General Solutions of HomogEqn, Review, Solving	4.1-4.4	4.1.1-3,4.3.1-4,4.5.1-3
8	108	Phase Plane Technique and Stability	4.6 4.7	4.6.1-2, 4.7.1-4
9	122	Nonlinear Systems and Stability	5.1 5.2	5.1.1-2, 5.2.1-2
10	138	Periodic Solutions and Central Force Field	5.3 5.4	5.3.1-3
11	155	Laplace Transform, Definition, Inverse, Properties, Solving DFQ	6.1-4	

3 Course Outline

- Chapter 1, Introduction
- Chapter 2, First Order Equations:
  Separable Equations, Integrating Factor, Modeling, Phase Plane Technique
- Chapter3. Second Order Equations:
  IVP, Homogeneous Equations, Inhomogeneous equations, Method of Undetermined Coefficients, Variations of a Parameter, Oscillations, Euler equation
- Chapter 4 Linear Systems: $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t)$.
- Chapter 5 Nonlinear Systems
- Chapter 6 Laplace Transforms:
  definition, inverse Laplace Transform, solving the ODE with Laplace transform
- Chapter 7 Partial Differential Equations:
  separation of variables, Sine and Cosine transforms, Wave Equations, BVP.

Here the problem are marked by a section number followed by a problem number. For example 2.2.1-3 means problems one, two and three in section 2.2.

4 Grade Policy, Homework, Exams

- Take all your quizzes, remove the worst one, average the result.
- Take your three midterms and quiz averages computed above, remove the worst number, average the result.
- Take the resulting number, multiply it by 0.55, add your Homework grade, multiiplied by 0.15 and add your final, multiplied by 0.3.
- Alternatively, take the resulting number, multiply it by 0.65, add your homeworks, multiplied by 0.15 and add your final, multiplied by 0.2.
- Your numerical grade is the larger of the two numbers.
- *Extra Credit : Final Project, 10 percent, limited availability.*

The final grade will be assigned based on the following:
$90 \leq g \leq 100$ is A
$80 \leq g < 90$ is B
$70 \leq g < 80$ is C
$64 \leq g < 70$ is D
$g < 66$ is F

Grades modifiers will be used: $0 \leq g \leq 3$ is "-", $3 < g \leq 7$ has no modifier, $7 < g$ is "+". Homeworks and exams may contain problems for extra-credit.

There is no "make up" policy for homeworks or quizzes.

Midterm make up policy If you miss one midterm, you will get grade zero for that midterm, and that would be the grade that is removed from your score as described in the grade policy. The make up policy for the second missed midterm is a Russian-style oral exam on the material covered. You are randomly given one paragraph title to present, and, after given time to prepare (20 minutes or so), you present the material of that paragraph to the professor. You are asked questions to probe the level of your understanding, and then you are given a problem to solve.

5 Few Additional Notes

5.1 Homeworks, attendance

There is no "curve grading".

Attendance may be be logged.

There is no requirement to attend class. This said, however, long-time experience shows that students who do not attend class and/or recitations usually do poorly in the course. Neither the instructor nor the teaching assistant are in any way responsible for briefing students who missed class on the missed material and/or announcements.

you are responsible for attending quizzes

Usage of **LAPTOPS, CELL PHONES, ipods or lightsabers.** will result in 5 percent substracted from your final grade.

My promise: Those who will pass the course will know Differential Equations

Academic Integrity The grade you receive for the course will be based on the work that you do.

With this principle in mind, the work (exams, homework, computer programs) that you present for a grade MUST, in fact, be of your own.

With respect to the exams, this means that no assistance or collaboration of any kind is permitted (other then assisace obtained from the instructor). Anyone violating this policy will receive an exam grade of zero and will be reported to the Dean of Students.

With respect to problem sets, you are free to seek assistance or advise from any person, book or computer. However, what you hand in must be your own work. Violating of this policy will result in a score of zero for the assignment and will be reported to the Dean of Students.

Second violation will get you grade F for the course.

Note that your health, need of financial aid, need to maintain GPA, need to graduate, obtain employment, etc **will not** be considered.

If you have serious problems when trying to solve the suggested homeworks, please seek tutoring from the Advising and Learning Assistance Center.

5.2 Drop In Tutoring

Advising and Learning Assistance Center holds free drop-in tutoring for Differential Equation Class. Contact Sharon McGrath, mcgras@rpi.edu.

6 Books

Introduction to Differential Equations 2e Second Edition, Mark Holmes,
ISBN-10 1975077202 ISBN-13 978-1975077204
Electronic Access Code ISBN: 0138265496 / 9780138265496 Net price $ 50
MyLab Math with eText Student Access Code for Fundamentals of Differential Equations with Boundary Value Problems Digital Update for RENSSELAER POLYTECHNIC INSTITUTE

- Go to https://mlm.pearson.com/enrollment/lvov75959.
- Sign in with your Pearson student account or create your account. For Instructors creating a Student account, do not use your instructor credentials.
- Select any available access option, if asked.
- Enter a prepaid access code that came with your textbook or from the bookstore.
- Buy instant access using a credit card or PayPal.
- Select Get temporary access without payment for 14 days.
- Select Go to my course.
- Select Math 2400 Fall 2024 from My Courses.
- If you contact Pearson Support, give them the course ID: lvov75959

To sign in later:

- Go to https://mlm.pearson.com.
-
- Sign in with the same Pearson account you used before.
-
- Select Math 2400 Fall 2024 from My Courses.

7 Final Project

Optional final project is consists of inclass 15 minutes presentation and a final written report on the topic of your choice where you use methods of differential equation to solve some cool problem.
Time line for final project:

- **stage one** - you think about what you want to do, talk to your friends, talk to me, bounce ideas of the wall - **September 7, 2024**.
- **stage 2**- you submit to me a title and a one page description of your project **September 15, 2024** to me.
- **stage 3** I choose 3 or 4 best projects and will notify you if you are selected.
- **stage 4** - you present to me your talk October 15, 2024
- **stage 4** you give in-class presentation and answer questions November 15, 2024
- Last day of classes - you give me your written report in LATEX format December 8, 2024

Submission of your final report constitutes your agreement to wave all the copyrights of your report. It may be made available to other DFQ students. To be eligible for a final project, your grade with out final project should be C or above

Welcome A

2

DFQ

SMILE

Lecture is recorded

yurilvov.com/DFQ

Differential

equations

everybody needs 2 or more ind. DEQ

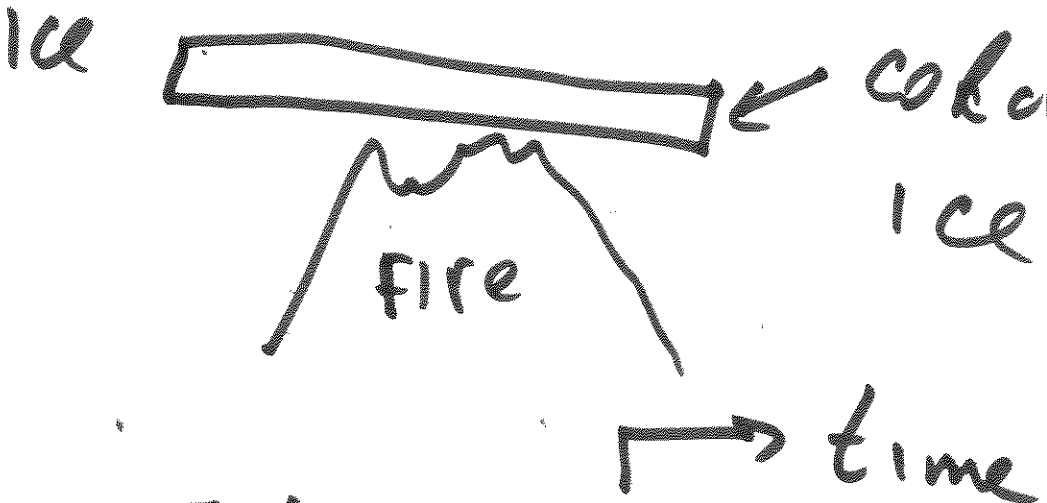
- PDE partial differential
- ODE ordinary — " —

one independent equations

- independent variable
- dependent variable

$$\frac{d}{dx} y(x) = ey(x) \quad \text{- ODE}$$

- $y(x)$: dependent variable
- x: independent



$T(x, t)$

↳ position

$$\frac{\partial}{\partial t} T(x,t) = \frac{\partial^2}{\partial x^2} T(x,t)$$

Single equation has one "=" sign

System of equations two or more "=" equations

$$\frac{d^2}{dt^2}x(t) + x(t) = 0$$

single

$$\begin{cases} x'(t) = y(t) \\ y'(t) = -x(t) \end{cases}$$

these are equivalent

- ORDER of Differential equation = highest derivative

first: $y'(x) = y(x)$

second $y''(x) + y(x) = 0$

third : $y'''(x) + y(x) = 0$

- Linear and non linear
  - dependent variable
    - $y(x) = (y(x))^1$
    - $(y(x))^a$ $a \neq 1$

Linear:

$$y'(x) + y(x) = 0$$ Is first order

$$x\,y'(x) + y(x) = 0$$ -"-

$$x^2 y''(x) + x\,y'(x) + y(x) = 0$$

Linear, 2nd order

non linear

$$y'(x) + y^2(x) = 0$$

$$\sqrt{y''(x)} = y(x)$$

Linear equations

homogeneous

non homogeneous

H: $y''(x) + y(x) = 0$

F

$y(x) = 0$ is not a solution

$y''(x) + y(x) = \cos(x)$

nonhomogeneous

because

$y(x) = 0$ is not a solution.

Solution transforms equation to equality

$$y'(x) = y(x)$$

$y(x) = e^x$; $\frac{d}{dx} y(x) = \frac{d}{dx} e^x = e^x$

$e^x = e^x$ ✓

$y(x) = C e^x$ - exact solution

↑ arbitrary constant

$$y''(x) + y(x) = 0$$

- $y(x) = A\cos x + B\sin x$

arbitrary constants

$$y'(x) = -A\sin x + B\cos x$$

- $y''(x) = -A\cos x - B\sin x$

$$-A\cos x - B\sin x + A\cos x + B\sin x =$$

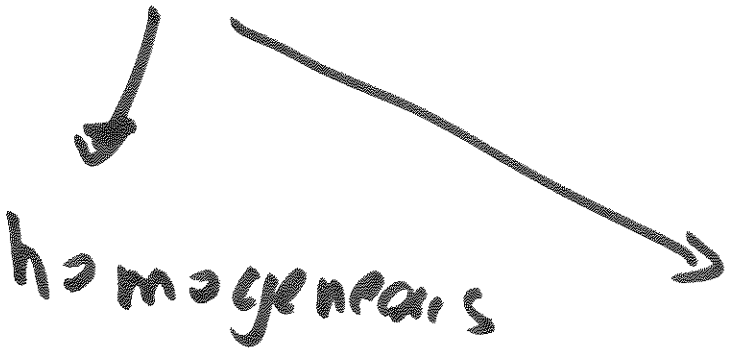
$$= 0$$

→ ODE or PDE

→ 1st order, 2nd order
any other integer
order

→ single, system

→ linear or nonlinear



homogeneous

non
homogeneous

- First order equations

$$\frac{d}{dt} y(t) = f(t, y(t))$$

For a given function $f(t, y)$

First order, nonlinear ordinary differential equation

no general equation solution

- Separable equation ~~f(t, y)~~

$$f(t, y(t)) = F(t) \cdot G(y(t))$$

$$f(t, y) = F(t)\, G(y) \checkmark$$

- IF $f(t,y)$ is linear in y, then there is an exact analytical solution

Ex $f(t,y) = a \cdot y + g(t)$

- a → constant
- $g(t)$ → some function

- IF $f(t,y) = F(y)$

$f(t,y)$ does NOT depend on t

Separable equations

$$\frac{dy}{dt} y(t) = \underbrace{F(t)\, G(y(t))}$$

Separable

* $\dfrac{dt}{G(y(t))}$ $\quad G(y(t)) \neq 0$

$$\left[\frac{dy}{dt} y(t) = F(t)\, G(y(t))\right] \frac{dt}{G(y(t))}$$

$$\frac{dy(t)}{G(y(t))} = F(t)\, dt \Big] \int$$

$$\int \frac{dy(t)}{G(y(t))} = \int F(t)\, dt$$

Implicit solution

Separable equations

• Assume $g(y) \neq 0$

• non unique

$$\left[\frac{d}{dx} y(x) = f(x)\, g(y(x))\right] dx$$

$g(y(x))$

$$\frac{d}{dx} y(x) = f(x)\, g(y(x)) \quad \times \frac{dx}{g(y(x))}$$

$$\int \frac{d\,y(x)}{g(y(x))} = \int f(x)\, dx$$

implicit solution

express $y(x)$ explicit solution

$y(t) = \ldots\ldots$

↑ explicit solution

Factorization may not be unique

$$2\,y(t)\cdot t^2 =$$

$$= (2y(t))\cdot t^2$$

$$= y(t)\cdot(2t^2)$$

Example 2

$$\frac{dy(t)}{dt} = y(t) \cdot t \; ; \quad y(t=0) = y_0 \; (*)$$

Initial condition

$$\int \frac{dy(t)}{y(t)} = \int t \, dt \qquad \text{implicit}$$

$$\ln y(t) = t^2/2 + C \qquad \text{implicit}$$

$$y(t) = e^C \cdot e^{t^2/2} \qquad \text{explicit soln}$$

$$y(t=0) = e^C = y_0$$

$$\text{Soln: } y(t) = y_0 \, e^{t^2/2}$$

$$y(t=0) = y_0 \; (*)$$

$$\begin{cases} 2\dfrac{dy(t)}{dt} = -y^3(t) \\ y(1) = 1 \end{cases}$$

ODE
1st order
nonlinear
separable

↳ Initial condition

$$-\int \frac{2\,dy(t)}{y^3(t)} = +\int dt$$

$$y(t) = \frac{1}{\sqrt{t+1}}$$

$$\frac{1}{y^2(t)} \Rightarrow y(t)^2 = t + c$$

$$y(t) = \frac{\pm 1}{\sqrt{t+c}}$$

$$y(t=1) = \frac{1}{\sqrt{c}} = 1 \Rightarrow c = 0$$

$$\frac{dy(t)}{dt} = \frac{y(t)}{1-y(t)} \qquad y(0)=1$$

$$\cdot \frac{1-y}{y}\, dt$$

$$\int \frac{dy\,(1-y)}{y} = \int dt$$

$$\ln y - y = t + C$$

Implicit

can not express $y(t)$ solution $= ?$ ...

$$-1 = C \Rightarrow C = -1$$

$$\ln y(t) - y(t) = t - 1$$

Example.

$$\frac{dy(x)}{dx} = -x/y(x) \quad \Big| \cdot y(x)\,dx$$

$$\int y(x)\,dy(x) = -\int x\,dx$$

$$y^2(x)/2 = -x^2/2 + C$$

$$y^2(x) + x^2 = 2C$$

$$y^2(x) + x^2 = 2C \quad \text{implicit solution}$$

$$y(x) = \pm\sqrt{2C - x^2}$$

↳ explicit solution

Integrating factor method

First order LINEAR differential equations.

$$y'(x) + p(x)\,y(x) = g(x)$$

where $p(x), g(x)$ are given functions

Integrating factor: $\mu(x)$

$$\mu(x)y'(x) + p(x)y(x)\mu(x) = g(x)\mu(x)$$

$$\frac{d}{dx}\left(\mu(x)y(x)\right) = \mu'(x)y(x) + \mu(x)y'(x)$$

$$\mu(x)y'(x) + p(x)y(x)\mu(x) = g(x)\mu(x)$$

$$= \frac{d}{dx}(\mu(x)y(x)) = \mu'(x)y(x) + \mu(x)y'(x)$$

$$\mu'(x)y(x) = p(x)y(x)\mu(x)\,;$$

$$\mu'(x) = p(x)\mu(x)$$

$$\int \frac{d\mu(x)}{\mu(x)} = \int p(x)\,dx$$

$$\mu(x) = e^{\int p(x)\,dx}$$